

Error Detection and Correction

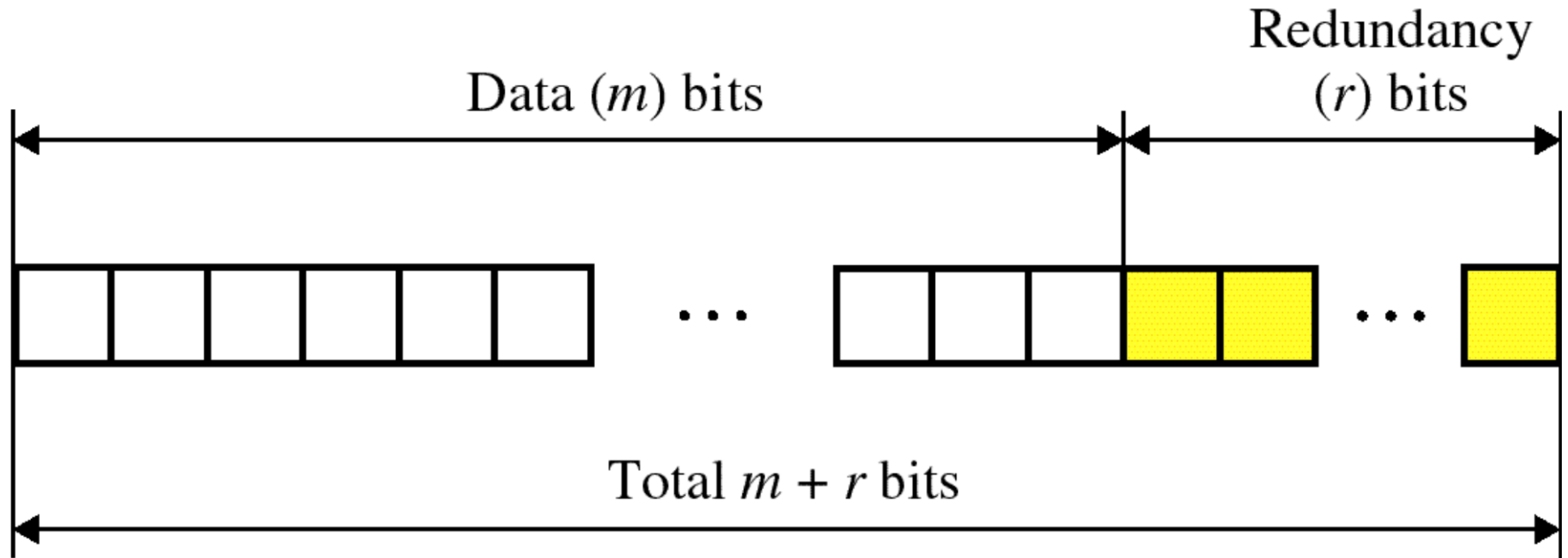


By

Dr M. Senthilkumar
Assistant Professor

Department of Computer Science
Government Arts and Science College, Avinashi - 641654

Error Correction



Error Correction – Hamming Code

✓ Consider 8 bit Data Word – 11000100

✓ Bit Position: 1 2 3 4 5 6 7 8 9 10 11 12

P1 P2 1 P4 1 0 0 P8 0 1 0 0

✓ The bits positions that are expressed in power of 2 are considered as Parity Bits

Error Correction – Hamming Code

$$\checkmark 1 = 2^0$$

$$\checkmark 2 = 2^1$$

$$\checkmark 3 = 2^0 + 2^1$$

$$\checkmark 4 = 2^2$$

$$\checkmark 5 = 2^2 + 2^0$$

$$\checkmark 6 = 2^2 + 2^1$$

$$\checkmark 7 = 2^2 + 2^1 + 2^0$$

$$\checkmark 8 = 2^3$$

$$\checkmark 9 = 2^3 + 2^0$$

$$\checkmark 10 = 2^3 + 2^1$$

$$\checkmark 11 = 2^3 + 2^1 + 2^0$$

$$\checkmark 12 = 2^3 + 2^2$$

$$\checkmark P1 = \text{XOR} (3, 5, 7, 9, 11)$$

$$\checkmark P2 = \text{XOR} (3, 6, 7, 10, 11)$$

$$\checkmark P4 = \text{XOR} (5, 6, 7, 12)$$

$$\checkmark P8 = \text{XOR} (9, 10, 11, 12)$$

Error Correction – Hamming Code

✓ Consider 8 bit Data Word – 11000100

✓ Bit Position: 1 2 3 4 5 6 7 8 9 10 11 12

P1 P2 1 P4 1 0 0 P8 0 1 0 0

$$P1 = \text{XOR}(3,5,7,9,11) = 1 + 1 + 0 + 0 + 0 = 0$$

$$P2 = \text{XOR}(3,6,7,10,11) = 1 + 0 + 0 + 1 + 0 = 0$$

$$P4 = \text{XOR}(5,6,7,12) = 1 + 0 + 0 + 0 = 1$$

$$P8 = \text{XOR}(9,10,11,12) = 0 + 1 + 0 + 0 = 1$$

Error Correction – Hamming Code

✓ Consider 8 bit Data Word – 11000100

✓ Bit Position: 1 2 3 4 5 6 7 8 9 10 11 12

0 0 1 1 1 0 0 1 0 1 0 0

$$P1 = \text{XOR}(3,5,7,9,11) = 0$$

$$P2 = \text{XOR}(3,6,7,10,11) = 0$$

$$P4 = \text{XOR}(5,6,7,12) = 1$$

$$P8 = \text{XOR}(9,10,11,12) = 1$$

Error Correction – Hamming Code

✓ Bit Position: 1 2 3 4 5 6 7 8 9 10 11 12

Transmitted: 0 0 1 1 1 0 0 1 0 1 0 0

Received: 1 0 1 1 1 0 0 1 0 1 0 0

$$C1 = \text{XOR}(1,3,5,7,9,11) = \text{Wrong } 1 + P1 = 1 + 0 = 1$$

$$C2 = \text{XOR}(2,3,6,7,10,11) = 0 + P2 = 0 + 0 = 0$$

$$C4 = \text{XOR}(4,5,6,7,12) = 1 + P4 = 1 + 1 = 0$$

$$C8 = \text{XOR}(8,9,10,11,12) = 1 + P8 = 1 + 1 = 0$$

Error in Bit 1



Error Correction – Hamming Code

✓ Bit Position: 1 2 3 4 5 6 7 8 9 10 11 12

Transmitted: 0 0 1 1 1 0 0 1 0 1 0 0

Received: 0 0 1 1 0 0 0 1 0 1 0 0

$$C1 = \text{XOR}(1,3,5,7,9,11) = 0 + \text{Wrong } P1 = 1 + 0 = 1$$

$$C2 = \text{XOR}(2,3,6,7,10,11) = 0 + P2 = 0 + 0 = 0$$

$$C4 = \text{XOR}(4,5,6,7,12) = 1 + \text{Wrong } P4 = 1 + 0 = 1$$

$$C8 = \text{XOR}(8,9,10,11,12) = 1 + P8 = 1 + 1 = 0$$

Error in Bit 5



Error Correction – Hamming Code

If $C = 0$ and $P = 0$, No Error

If $C \neq 0$ and $P = 1$, Single Bit Error, Can be Corrected

If $C \neq 0$ and $P = 0$, Double Bit Error, Can be Detected, Can not be Corrected

If $C = 0$ and $P \neq 0$, P13 Bit Error

References

- ✓ Book: Data communication and Networking
Fourth edition
By : BEHROUZ A FOROUZAN
- ✓ various relevant websites

Thank You